The Tactics about Orbit Determination of Satellite with the Short-Arc Tracing Data

Jing He, Zhen-dong Xi, and Yang Liu

***Abstract*—The orbit determination not only is the one of the main ways to evaluate the precision class of controlling the craft launching and separating satellite from rocket,but also the resource of guiding data for later observations.These different kinds of orbit differ greatly in field angle to the earth's core in equal time interval.So that will produce different effects on the result of orbit determination. Two projects are offered for a high-accuracy orbit determination in define observation data arc except choicing a site where the observation data arc may cover the perigee.One way is to lengthen data arc.The other is to add data kinds.Last those three methods are introduced and analysed in view of those two means.**

***Index Terms*—Orbit determination tactics, short –arc data arc, lauch into orbit, TTandC data**

I. Introduction

In past several years ,many kinds of method of orbit determination were put forward,such as about tracking and data relay satellite system, navigation satellite, computing orbit method, orbit transfer and so on.But the research on using short-arc data to position is rarely, much less method about using short-arc data in orbiting a satellite.

The result of initial orbit in orbiting arc is one of main keys to value the control precision of lauching vehicles and separation of the spacecraft from the launcher.That is also the homing data for later observation station.But there is only very short time TTandC data can be used to determine orbit in lauching vehicles into orbit without artificial impetus.So it is hard to get a high level of accuracy result.Even it is hard to come out a result sometimes in virtue of that course of calculating is not convergent.And there is maybe different in equal time intervals.For example,using the same 30 or 40 secnds data , it is easy and fast for airship kind of assignment to get a good outcome and difficult in exploring-1 assignment with inconsistent orbit elements. The result precision of short hour is higher than that of long hour with same error data and well-working equipment at whiles.

It is analysed that different kinds of orbit is provided with different earth's core angle[1]. So the observation station will chose the position near the perigee to the best of one's abilities If in same measuring hour refer to article[1]. If the measuring arc has been choosen only to find out some other ways to do for us. In this paper two kinds of project are provided.One way is to lengthen the data arc.The other is to add data variety in limited data arc.

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II. The Analysis of Different Sorts of Orbit in Different Measuring Position

A. Velocity and ItsVariety Rule in Orbital Motion Ellipse Plane

Base on classical orbital laws,we could infer the angular velocity and linear velocity in orbtal plane:

$$\dot{\theta} = \frac{\sqrt{\mu a(1-e^2)}}{r^2}, \quad v^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right) .$$

These two equations implies craft planetary motion is not symmetrical,in other words, moves fast near the perigee and slow near the apogee..

B. Angle to the Earth's Core and its Mutative Rule

Due to the asymmetry, the path of satellite circulating in equal time intervals at different position is corresponding different true anomaly in orbit plane. So four different kinds of orbit are simulated at the following text including rule of true anomaly varies with mean anomaly(fig.1) and the size of true anomaly at different mean anomaly with 30,60 or 120 seconds(fig.2).For simplification only the force of two-body model is considered here and other disturbing forces are ignored.

TABLE I: Orbit Size and Orbit Shape

serial number	a（km）	e
1	6648.189	0.01080
2	27473.640	0.76056
3	46130.162	0.84692
4	67054.000	0.89593

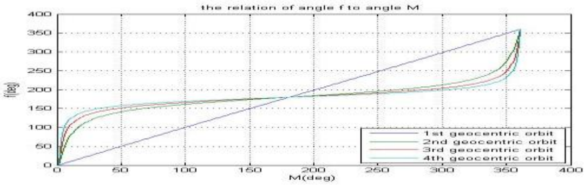

Fig. 1. The relation between eccentric anomaly e and true anomaly f in different orbit.

From Fig.1, true anomaly keeps ahead in（180 °, 360 °) and lag in(180 °, 360 °)comparing with mean anomaly.That the field angle becomes larger with longer time is found due to the fig. 2, especially near the perigee in big orbital eccentricity plane.Also the size of the field angle becomes little in equal time interval in small orbital eccentricity plane.But only if the tracking arc is near or cover the perigee ,the size of that will be biggish in big orbital eccentricity plane.So the accuracy of orbit determination is also improved if using the data in tracking arc more near or cover the perigee for that kind of big orbital eccentricity.

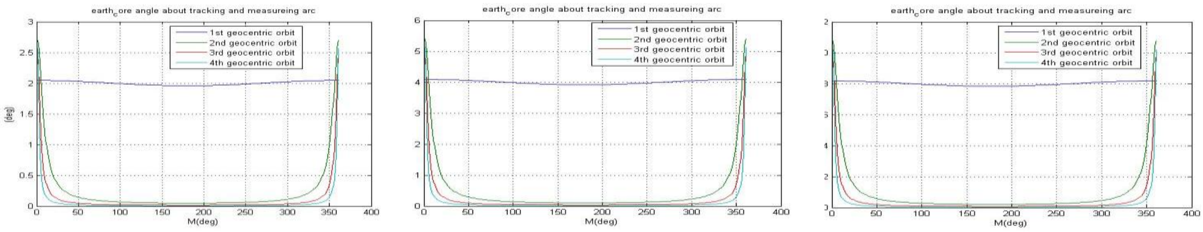

Fig. 2. The field angle to the earth's core with 30(left), 60(center), 120(right) Seconds data at different mean anomaly

III. METHODS OF COMBINED ORBIT DETERMINATION WITH UNITED DATA IN ORBITING

Two plans were brought forward aiming at how to add data to be used in orbital determination before orbit when there is a little TTandC data to be utilized.One is to extending usable data arc.The other is to add data sorts in finite time interval.

A. Method of Orbit Determination with United TTandC Data from Rocket

Satellite and rocket have same pace because they are combined from the time when stopping modifying velocity of rocket to the time when separating satellite from rocket. There is only position difference for cooperative TTandC target between in rocket and in satellite.

If retro-rocket was chosen continuous data from satellite before or after separation will theoretically be used to compute orbital elements.But data from rocket after separation is not commonly used on account of that the particular thrust force adeffect of rocket engine and its duration are hard to get.

If eject manner was chosen in separation and no other drive but gravitation or no TTandC data from satellite we may think about introducing the method of associating with data before separation and after separation.The idea is that amending data from rocket after separation first ,then computing the academic orbit before separation,at last computing satellite orbit after separation by correcting velocity and position increment at the time of separation. Commonly only velocity increment is taken into account because of magnitude of position increment less than that of velocity increment out and away in the course of calculating orbit with standard units pattern.

- Data Amending from Rocket after Separation.

The key in the course of amending is that data and increment are in same reference frame by conversion for data from rocket after separation. Increment parameter is usually presented in lauching reference frame.So found $(R \quad E \quad A)_i$ or $(x \quad y \quad z)_{1i}^T$ as TTandC data in horizon reference frame, $(x \quad y \quad z)_{2i}^T$ as data in lauching inertial reference frame, $(x \quad y \quad z)_{3i}^T$ as data in lauching inertial reference frame after correction, $(x' \quad y' \quad z')_{1i}^T$ or $(R' \quad E' \quad A')_i$ as data in horizon reference frame after correction。t_i, t_f are respectively the time of data itself and that of separation，$(t_i - t_f)$ takeτ（τ=806.8116s）as unit. $(vx \quad vy \quad vz)_1^T$ is the velocity increment named $\Delta v1$ getted from rocket in lauching reference frame.i=1,2,…,n.

$$(x \quad y \quad z)_{2i}^T = F(x \quad y \quad z)_{1i}^T ,$$

$$(x \quad y \quad z)_{3i}^T = (x \quad y \quad z)_{2i}^T + (vx \quad vy \quad vz)^T \cdot (t_i - t_f) ,$$

$$(x' \quad y' \quad z')_{1i}^T = F^{-1} \ (x \quad y \quad z)_{3i}^T .$$

Matrix F is the conversion matrix from standard horizon reference frame to lauching inertial reference frame. Whereas the conversion described in some textbook too much that is not expressed here.

- Orbit the time of t_f with data before separation connecting with that after separation.
- Correct Velocity Increment.

First chang orbit element into position vector $(x \quad y \quad z)_f^T$ and velocity vector $(\dot{x} \quad \dot{y} \quad \dot{z})_f^T$ at the time of t_f .Then convert $(vx \quad vy \quad vz)_2^T$ which is the coordinate of $\Delta v2$ getted by satellite in lauching inertial reference frame into $(vx \quad vy \quad vz)_{2g}^T$ in the earth's core inertial reference frame as illustrated in formula down.Finally turn and $(\dot{x} \quad \dot{y} \quad \dot{z})_f^T{}'$ into orbital elements. $(\dot{x} \quad \dot{y} \quad \dot{z})_f^T{}' = (\dot{x} \quad \dot{y} \quad \dot{z})_f^T + (vx \quad vy \quad vz)_{2g}^T$.

B. Method of Orbit Determination with United TTandC Data from Rocket

Qualification expression is newly defined for orbiting with kinds of data in same pace,such as PUVM1[2].（ρ E A）is TTandC data, $\vec{r} = (x \quad y \quad z)^T$ *and* $\dot{\vec{r}} = (\dot{x} \quad \dot{y} \quad \dot{z})^T$ are remote sensing data.Its qualification expression refer to formula(1).

$$\begin{cases} f\vec{\rho}^* \cdot \vec{r}_0 + g\,\vec{\rho}^* \cdot \dot{\vec{r}}_0 = \vec{\rho}^* \cdot \vec{R} - \omega\vec{\rho}^* \cdot \vec{W}_0 + \rho \\ f\vec{h}^* \cdot \vec{r}_0 + g\vec{h}^* \cdot \dot{\vec{r}}_0 = \vec{h}^* \cdot \vec{R} - \omega\vec{h}^* \cdot \vec{W}_0 \\ f\vec{A}^* \cdot \vec{r}_0 + g\vec{A}^* \cdot \dot{\vec{r}}_0 = \vec{A}^* \cdot \vec{R} - \omega\vec{A}^* \cdot \vec{W}_0 \\ f \cdot \vec{r}_0 + g \cdot \dot{\vec{r}}_0 = \vec{r} - \omega\vec{W} \\ \dot{f} \cdot \vec{r}_0 + \dot{g} \cdot \dot{\vec{r}}_0 = \dot{\vec{r}} - \dot{\omega}\vec{W} \end{cases} \tag{1}$$

$\dot{f}, \dot{g}, \dot{\omega}$ are defined in the paper [3].

C. Syncretize Method of Orbit Determination with Kinds of Data in Orbiting Based on Vector Decompounding Aslant[4]

That the syncretize method of kinds of Data in same pace is anew deduced based on the differential coefficient orbital determination. Formula(2)is its qualification expression . TTandC data and remote sensing data are defined ditto.

$$\begin{cases} A_o - A_c = A_{\vec{r}_c} M_{\vec{r}} \Delta\vec{s}_0 \\ E_o - E_c = E_{\vec{r}_c} M_{\vec{r}} \Delta\vec{s}_0 \\ \rho_o - \rho_c = \rho_{\vec{r}_c} M_{\vec{r}} \Delta\vec{s}_0 \\ x_o - x_c = M_{\vec{r}} \Delta\vec{s}_0 \\ y_o - y_c = M_{\vec{r}} \Delta\vec{s}_0 \\ z_o - z_c = M_{\vec{r}} \Delta\vec{s}_0 \\ \dot{x}_o - \dot{x}_c = M_{\vec{r}} \Delta\vec{s}_0 \\ \dot{y}_o - \dot{y}_c = M_{\dot{\vec{r}}} \Delta\vec{s}_0 \\ \dot{z}_o - \dot{z}_c = M_{\dot{\vec{r}}} \Delta\vec{s}_0 \end{cases} \quad (2)$$

And $A_{\vec{r}_c} = \left(\frac{\partial A_c}{\partial \vec{r}_c}\right)^T, E_{\vec{r}_c} = \left(\frac{\partial E_c}{\partial \vec{r}_c}\right)^T, \rho_{\vec{r}_c} = \left(\frac{\partial \rho_c}{\partial \vec{r}_c}\right)^T$,

$\Delta\vec{s} = (\Delta x, \Delta y, \Delta z, \Delta\dot{x}, \Delta\dot{y}, \Delta\dot{z})^T$,

$M_{\vec{r}} = (fE_{3\times3} \quad gE_{3\times3}) + (wGrad(W_0))$,

$M_{\dot{\vec{r}}} = (\dot{f}E_{3\times3} \quad \dot{g}E_{3\times3}) + (\dot{w}Grad(W_0))$,

$Grad(\vec{W}_0) = (\frac{\partial W_0}{\partial x}, \frac{\partial W_0}{\partial y}, \frac{\partial W_0}{\partial z}, \frac{\partial W_0}{\partial \dot{x}}, \frac{\partial W_0}{\partial \dot{y}}, \frac{\partial W_0}{\partial \dot{z}})$.

IV. Results and Analysis References

Results of orbit determination in X1 assignment with different data arc are calculated and minus standard elements.Then those are filled in tab2.One data arc is 80 seconds before separation.The other is 80 seconds before separation and 60 seconds after separation.Results of orbit determination in X2 , X3 assignment with TTandC and remote sensing data in same period of time are solely calculated by means of 2nd method and 3rd method.Then they are minus standard elements.The difference is infilling in tab.2.

In tab.2 the old method is the method according to article[2] by 80 seconds data before separation.And 1st method, 2nd method and 3rd method are three methods of orbit determination with united data in orbiting introduced in section3.2.

From the results above,it is obvious that results of later three orbit determination methods improved dissimilarly compared with the old.And results of 1st method is slightly better than the old .And 3rd method is better than 2nd method.The main reason is the better weight of radial rate with high class of precision expatiated in article [4] in detail.

TABLE II: Computing Resasults

resource	method	⊿a（km）	⊿e(10^{-4})	⊿i(°)	⊿Ω(°)	⊿ω(°)	⊿M(°)
X1	Old method	0.862	0.0883	0.0029	0.0739	0.0079	0.0016
	1st method	0.764	0.0783	0.0021	0.0743	0.0075	-0.0009
X2	old method	-4.442	-0.0395	0.0023	-0.53300	-0.02489	0.002169
	2nd method	1.297	0.1310	-0.03781	-0.55508	0.0227	-0.00026
	3rd method	-0.225	-0.0690	-0.03804	-0.54963	0.0280	-0.00202
X3	Old method	-1.8249	0.01198	-0.04163	0.013949	-0.08717	0.007547
	2nd method	0.368	0.0303	-0.05125	0.007765	-0.00673	-0.00031
	3rd method	0.168	0.0166	-0.05126	0.008052	-0.00815	-0.00009

1st method improve the accuracy of orbit determination by extending congenericdata arc and that may effectively minish random error. 2nd method and 3rd method both add data species in equal time interval. As a rule those may both improve the accuracy of orbit determination and escape extreme result.Thus it is good at stability.Especially 3rd method may obviously meliorate the result of orbit determination in virtue of rational weight for multiplicate data.But whether 2nd method and 3rd method may improve the accuracy of orbit determination or not,it needs to analyse concretely error distributing of each data.It is explaied some in paper[5].And choosing data arc still needs analyse concretely the motion of spacecraft before separation,such as whether small force exist or not, whether pose is adjusted or not and so on.So that part is not the emphases researched on here and say more than is needed anymore.

The precision of orbit determination in orbiting satellite is one of important means to evaluate the accuracy of launching vehicle.So the accuracy of orbit determination with short arc in orbiting has been more paid attention to. What we can do to improve the precision of orbit determination is that we may add data by extending data arc or adding data species in limit time besides chosing the position of observation station in measuring data arc at the time of designing the plan that where and how to track and measure the spacecraft. Compared with the old method those three methods here all have better result to some extent. The paper [6] has mentioned the betterment of method for extracting in estimate data error. Similarly that whether the root of the equation in combined orbit determination exist or not relates with the system error and its distributing.So we will lay stress on the error analysis of different kinds of data and corresponding amelioration next.

References

[1] J. He, B. Liu, and Y. Liu, “The Analysis for the Effect of different Observation arc on Orbit Determination,” The 22nd conference about flight dynamics and air tria hold by china Astronautica institute. Sichuan CHENGdou. 2006, pp. 572-575.

[2] B. K. Lu, J. F. Li, and J. Y. Ma, et al. “A method of perturbed initial orbit determination,” *journal of astronautics*. 1999, vol. 20, no. 1, pp. 14-20.

[3] J. He, J. Y. Ma, and B. K. Lu, et al. “The Analysis for the Effect of Motion of Observation Station on Orbit Determination,” *Acta Astronomica Sinica*. 2005, vol. 46, no. 2, pp. 181-185.

[4] W. S. Chen, J. Y. Ma, and J. Zhang, *et al.*, “Approximate-State Transfer Matrix of Vector decompose resolved in lean direction and it’s application in orbit determination,” *Science in China(Series G:Physics,Mechanics and Astronomy)* 2009, vol. 39, vol. 11, pp. 1664-1670.

[5] Y. X. Mao, X. Q. Ning, and ZH. H. Zhang, *et al.*, "Analysis of Orbit Determination Precision with Data in Attitude Adjusting Segment before Separation of Satellite-rockets in Space launch Mission," *Journal of Spacecraft TTandC Technology*. 2010, vol. 29, no. 2, pp. 44-49.

[6] Q. Gao, Y. Wang, and X. J. Du, *et al.*, "Ill-condition and registration of system error estimation in data fusion systems," *Journal of Tsinghua University(Science and Technology)* 2010 , vol. 50, no. 4, pp. 591-594.